ONE-FLUID DYNAMICAL EQUATIONS OF A PARTIALLY IONIZED GAS IN A STRONG MAGNETIC FIELD

(ODNOZHIDKOSTNYE URAVNENIIA DINAMIKI CHASTICHNO IONIZOVANNOGO GAZA V SIL'NOM MAGNITNOM POLE)

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E.G.SAKHNOVSKII

(Leningrad)

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It is well known that the Larmour rotation of charged particles in a strong magnetic field leads to anisotropy of transport phenomena in an ionized gas. For this, the methods of the mechanics of continuous media do not give information about the viscous stress tensor and the heat flux vector in the principles of conservation of momentum and energy [1]. That information can be obtained by the methods of statistical mechanics; thus, in the last few years, there have appeared a number of papers which use the kinetic theory of gases to investigate transport phenomena in a plasma. Using various assumptions, references [2 to 8] obtain systems of equations describing the behavior of a fully ionized plasma in a magnetic field, either in the two-fluid approximation, in which the plasma is represented by interpenetrating ion and electron gases, or in the one-fluid model in which the mixture is treated as a whole. Paper [9] is devoted to the influence of neutral particles, having a Maxwellian distribution, on the transport processes in a uniform magnetic field. Finally, in [10 and 11] in the "13-moment" approximation in Grad's method, a closed system of equations for transport in a magnetic field is found, separately for each of the electron, ion, and neutral gases making up the partially ionized plasma.

In what follows, a closed system of equations is constructed to describe the behavior of such a plasma as a whole.

From this system, the well-known equations of isotropic magnetohydrodynamics, as well as the one-fluid equations of dynamics of a fully-ionized gas, arise as particular cases. Also, the results obtained make it possible to investigate other limiting cases, for example, the flow of a weakly ionized gas.

As in [10], the following assumptions are made:

1) A monatomic gas is considered.

2) The interaction between particles (including Coulomb interaction) is described in terms of pair collisions.

3) Phenomena connected with nonelastic collisions are not taken into account.

4) The macroscopic parameters of the gas change hardly over distances of the order of the mean free path, and over a time interval of the order of the time between collisions.

5) $T_e = T_i = T_a = T$; here, T_α is a temperature, index α indicates the component (electron, ion or neutral particle).

6) It is assumed that $\sqrt{m_e / m_i} \ll 1$, where m_α is the mass of the α-particle.

7) The gas is assumed to be electrically quasi-neutral, i.e. $n_e e_e + n_i e_i + n_a e_a \approx 0$, where n_α and e_α are, respectively, the partial number density and the charge of the α-component.

Putting $e_i = -Ze_e = Ze$, we have the condition for the concentration, $n_e \approx Zn_i$.

The possibility of the existence of a small volume charge is included in Maxwell's equations by div $\mathbf{D} = \rho_e$, where $\rho_e = e(Zn_i - n_e)$ is the volume charge, Z is the charge number, $\mathbf{D}$ is the electric induction vector.

1. Viscous stress tensor. It is well known [12] that, for an isotropic medium, in the absence of a magnetic field, the viscous stress tensor π^{rm} is simply related to the rate of strain tensor

$$\pi^{rm} = -\eta e^{rm}, \qquad e^{rm} = \frac{\partial u^r}{\partial x_m} + \frac{\partial u^m}{\partial x^r} - \frac{2}{3}\delta^{rm}\frac{\partial u^e}{\partial x_e} \tag{1.1}$$

Here $\mathbf{u}$ is the mean mass velocity of the gas, δ^{rm} is the Kroncker symbol, and the scalar coefficient η is called the viscosity coefficient. For an anisotropic medium, this relation becomes more general, remaining linear since departures from equilibrium are considered small.

The symmetric tensor e^{rm}, with trace equal to zero, has five linearly independent components, therefore, the general form of the linear homogeneous relation between π^{rm} and e^{rm} contains five independent coefficients of proportionality. It is natural to call them the coefficients of viscosity of such an anisotropic medium.

Following Braginskii [2], we introduce the tensors

$$\begin{aligned}
W_0^{rm} &= {}^3\!/_2\,(b^r b^m - {}^1\!/_3\,\delta^{rm})\,(b^\mu b^\nu - {}^1\!/_3\,\delta^{\mu\nu})\,e^{\mu\nu} \qquad (\mathbf{b} = \mathbf{B}/B)\\
W_1^{rm} &= (\delta_\perp^{r\mu}\delta_\perp^{m\nu} + {}^1\!/_2\,\delta_\perp^{rm} b^\mu b^\nu)\,e^{\mu\nu} \qquad (\delta_\perp^{rm} = \delta^{rm} - b^r b^m)\\
W_2^{rm} &= (\delta_\perp^{r\mu} b^m b^\nu + \delta_\perp^{m\nu} b^r b^\mu)\,e^{\mu\nu}\\
W_3^{rm} &= {}^1\!/_2\,(\delta_\perp^{r\mu}\varepsilon^{m\gamma\nu} + \delta_\perp^{m\nu}\varepsilon^{r\gamma\mu})\,b^\gamma e^{\mu\nu}\\
W_4^{rm} &= (b^r b^\mu \varepsilon^{m\gamma\nu} + b^m b^\nu \varepsilon^{r\gamma\mu})\,b^\gamma e^{\mu\nu}
\end{aligned} \tag{1.2}$$

Here $\mathbf{B}$ is the magnetic induction vector and $\varepsilon^{rm\gamma}$ is the permutation tensor. In addition,

$$W_0^{rm} + W_1^{rm} + W_2^{rm} = e^{rm}, \qquad W_k^{rm} W_n^{rm} = 0 \qquad \text{for } k \neq n \tag{1.3}$$

Then, solving Equations (2.1) and (2.6) of [10], and summing over all components, we will have, for the viscous stress tensor in an arbitrary Cartesian coordinate system, the expression

$$\pi^{rm} = -\eta^{(0)} W_0^{rm} - \eta^{(1)} W_1^{rm} - \eta^{(2)} W_2^{rm} + \eta^{(3)} W_3^{rm} + \eta^{(4)} W_4^{rm} \tag{1.4}$$

The five coefficients of viscosity can be expressed in terms of the partial coefficients of viscosity of the electrons, ions, and neutral particles

$$\eta^{(k)} = \sum_{\alpha = e,\, i,\, a} \eta_\alpha{}^{(k)} \qquad (k = 0, 1, 2, 3, 4) \tag{1.5}$$

The latter have the following form:

1) Coefficients of viscosity of electrons

$$\eta_e^{(0)} = \eta_e, \qquad \eta_e^{(1)}(\omega_e) = \eta_e^{(2)}(2\omega_e) = \frac{\eta_e^{(0)}}{1 + {}^{16}/_9\,\omega_e^2\tau_e^2}$$
$$\eta_e^{(3)}(\omega_e) = \eta_e^{(4)}(2\omega_e) = -\frac{{}^4/_3\,\omega_e\tau_e\eta_e^{(0)}}{1 + {}^{16}/_9\,\omega_e^2\tau_e^2} \tag{1.6}$$

2) Coefficients of viscosity of ions

$$\eta_i^{(0)} = (1 + \alpha\tau_a\tau_{ia}^{-1})\,\theta\eta_i, \qquad \eta_i^{(1)}(\omega\ ) = \eta_i^{(2)}(2\omega_i) = \frac{\eta_i^{(0)}}{1 + {}^{16}/_9\,\omega_i^2\tau_i^2\theta^2}$$
$$\eta_i^{(3)}(\omega_i) = \eta_i^{(4)}(2\omega\ ) = \frac{{}^4/_3\,\omega_i\tau_i\theta\eta_i^{(0)}}{1 + {}^{16}/_9\,\omega_i^2\tau_i^2\theta^2} \tag{1.7}$$

3) Coefficients of viscosity of neutral particles

$$\eta_a^{(0)} = (1 + \alpha\tau_i\tau_{ai}^{-1})\,\theta\,\eta_a, \qquad \eta_a^{(1)}(\omega_i) = \eta_a^{(2)}(2\omega_i) = \frac{\eta_a^{(0)} + {}^{16}/_9\,\omega_i^2\tau_i^2\theta^2\eta_a}{1 + {}^{16}/_9\,\omega_i^2\tau_i^2\theta^2}$$
$$\eta_a^{(3)}(\omega_i) = \eta_a^{(4)}(2\omega_i) = \frac{{}^4/_3\,\omega_i\tau_i\theta}{1 + {}^{16}/_9\,\omega_i^2\tau_i^2\theta^2}(\eta_a^{(0)} - \eta_a) \tag{1.8}$$

$$\theta^{-1} = 1 - \alpha^2\tau_a\tau_i\tau_{ia}^{-1}\tau_{ai}^{-1}, \qquad \eta_\alpha = {}^2/_3\,n_\alpha kT\tau_\alpha \qquad (\alpha = e,\ i,\ a)$$

Here, k is Boltzmann's constant, and the other notation is the same as in [10]. (*)

We shall represent the viscous stress tensor by means of one-fluid parameters. We shall express the particle number densities n_α appearing in η_α, τ_α, and $\tau_{\alpha\beta}$ by the mass density of the gas ρ and its degree of ionization s.

$$\rho = \sum_\alpha n_\alpha m_\alpha, \qquad s = \frac{n_i}{n_i + n_a}$$

Using the condition $n = \sum_\alpha n_\alpha$, for the number density of the mixture, we have

$$n_e = Zn_i = \frac{Zs}{1 + Zs}n, \quad n_a = \frac{1 - s}{1 + Zs}n, \quad n = \frac{1 + Zs}{Zsm_e + m_i}\rho \approx \frac{1 + Zs}{m_i}\rho \tag{1.9}$$

Thus, we find

$$n_e = Zn_i = \frac{Zs}{m_i}\rho, \qquad n_a = \frac{1 - s}{m_i}\rho \tag{1.10}$$

accurate up to terms of order m_e/m_i.

It is not difficult to see that there are three characteristic directions of momentum transport along which the gas viscosity is different. The first transport, along the magnetic field, of the component of momentum in the

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*) In [10] there is an error in the expressions for τ_α^{-1} and $(\tau_\alpha^*)^{-1}$. In the items $0.4\ \tau_{\alpha\alpha}^{-1}$ the factor $A_{\alpha\alpha}^*$ is missing.

direction of the field. It is characterized by the coefficient $\eta^{(0)}(T, s, \rho)$, which is equal to the viscosity of the partially ionized gas in the absence of a magnetic field. In the other two directions, characterized by the coefficients $\eta^{(1)}(T, s, \rho, B) = \eta^{(2)}(T, s, \rho, 2B)$ and $\eta^{(3)}(T, s, \rho, B) = \eta^{(4)}(T, s, \rho, 2B)$, the viscosity of the partially ionized plasma is strongly reduced in the presence of the field.

Let us now examine the viscous stress tensor for the particular cases of a weakly ionized and fully ionized gas. Putting $s \to 0$ in (1.5), we find, after simple calculations,

$$\eta^{(k)} = \begin{cases} [\eta_a]_{s=0} + O(s) & \text{for } k = 0, 1, 2 \\ O(s) & \text{for } k = 3, 4 \end{cases} \qquad \left([\eta_a]_{s=0} = \frac{5kT}{8\Omega_{aa}^{2}(2)} \right) \tag{1.11}$$

This coincides with the coefficient of viscosity of a simple gas according to Chapman-Cowling [12]. From this it follows that, for a weakly ionized gas, within the assumptions that have been made, the hydrodynamic viscous stress tensor can be used.

Before proceeding to the case of fully ionized gas, we note that, for arbitrary s, $\omega_e\tau_e$ and $\omega_i\tau_i\theta$, the contribution of the electrons may be omitted from the viscous strain tensor with accuracy up to quantities of order $\sqrt{m_e/m_i}$. This follows from an estimate of the ratios $\eta_e^{(k)} / \eta_i^{(k)}$ (k = 0, 1, 2, 3, 4). Thus, instead of (1.5), it is sufficient to take

$$\eta^{(k)} = \sum_{\alpha=i,\, a} \eta_\alpha^{(k)} \qquad (k = 0, 1, 2, 3, 4) \tag{1.12}$$

From this $\eta^{(k)} = \eta_i^{(k)}$ for $s = 1$ (see [2 and 7]). Then, putting $s = 1$ in (1.7) and writing (1.4) in the special coordinate system with z-axis parallel to $\mathbf{b}$, we obtain expressions which coincide with those given in [7 and 8] (up to numerical factors of the coefficients). The difference is due to different methods of solution of Boltzmann's equation [2 and 10], in particular, Grad's "13-moment" approximation in the second reference, and, in the first reference, development of the correction to Maxwell's function in a series of Sonin polynomials, taken up to two terms.

2. Heat flux vector. As may be seen from Equations (1.13), (3.3) and (3.8) of [10], in order to write down the heat flux vector of a partially ionized gas in the one-fluid approximation, it is necessary to have expressions for the diffusion velocities $\mathbf{w}_\alpha$ which make up its components. Using condition $\Sigma m_\alpha n_\alpha \mathbf{w}_\alpha = 0$ and the expression for the conduction flux $\mathbf{j} = \Sigma n_\alpha e_\alpha \mathbf{w}_\alpha = -n_e e(\mathbf{w}_e - \mathbf{w}_i)$, we have with accuracy to terms of order m_e / m_i

$$\mathbf{w}_e = -\frac{m_i}{Zes\rho}\mathbf{j} + (1-s)\mathbf{V}_i, \qquad \mathbf{w}_i = -\frac{m_e}{e\rho}\mathbf{j} + (1-s)\mathbf{V}_i$$

$$\mathbf{w}_a = -\frac{m_e}{e\rho} j - s\mathbf{V}_i \tag{2.1}$$

The ion "slip" velocity $\mathbf{V}_i = \mathbf{w}_i - \mathbf{w}_a$ is given by Expression (4.8) in [10], which we rewrite, using the one-fluid parameters and taking onto account the viscous term,

$$\mathbf{V}_i = \varepsilon \frac{m_i}{Zes\rho} [\mathbf{j} + (1 - s)\, \omega_e \tau_{ea} (\mathbf{j} \times \mathbf{b})] -$$
$$- \frac{2\tau_{ia}}{s\rho} \left[\frac{Z+1}{(1+Zs)^2} p \nabla s + \frac{Zs(1-s)}{1+Zs} \nabla p - s \operatorname{div} \pi + \operatorname{div} \pi_i \right] - \tag{2.2}$$
$$- \frac{1+Zs}{p} \left[\varepsilon \zeta_{ea} \frac{\mathbf{h}_e}{Zs} + \frac{1}{2} \zeta_{ia} \left(\frac{\mathbf{h}_i}{s} - \frac{\mathbf{h}_a}{1-s} \right) \right] \qquad \left(\varepsilon = 2Z \frac{m_e \tau_{ea}^{-1}}{m_i \tau_{ia}^{-1}} \sim Z \left(\frac{m_e}{m_i} \right)^{1/2} \ll 1 \right)$$

Here p is the pressure, $\mathbf{h}_\alpha$ is the heat flux vector corresponding to the α-component [10].

Then, neglecting the influence of viscosity on the heat flux, using the assumptions that have already been made, and taking into account the observations made in [10] concerning the terms in (2.2), we can obtain from (1.13), (3.4) and (3.5) of paper [10] the following general expression for the heat flux vector:

$$\begin{aligned} \mathbf{q} = & - \lambda^T \nabla T - \lambda^{T\parallel} \mathbf{b} (\nabla T \cdot \mathbf{b}) - \lambda^{T\perp} (\nabla T \times \mathbf{b}) - \\ & - (\lambda^p + \lambda_g^p) \nabla p - \lambda^{p\parallel} \mathbf{b} (\nabla p \cdot \mathbf{b}) - \lambda^{p\perp} (\nabla p \times \mathbf{b}) - \\ & - (\lambda^s + \lambda_g^s) \nabla s - \lambda^{s\parallel} \mathbf{b} (\nabla s \cdot \mathbf{b}) - \lambda^{s\perp} (\nabla s \times \mathbf{b}) - \\ & - (\lambda^j + \lambda_g^j)\, \mathbf{j} - \lambda^{j\parallel} \mathbf{b} (\mathbf{j} \cdot \mathbf{b}) - (\lambda^{j\perp} + \lambda_g^{j\perp}) (\mathbf{j} \times \mathbf{b}) \end{aligned} \tag{2.3}$$

The relative heat flow $\mathbf{h}$ is characterized by the coefficients of heat conductivity

$$\mathbf{h} = \mathbf{q} - \frac{5}{2} \sum_{\alpha=e,\, i,\, a} n_\alpha k T \mathbf{w}_\alpha \tag{2.4}$$
$$\lambda^k (T, s, \rho, B) = \sum_{\alpha=e,\, i,\, a} \lambda_\alpha^k \; (k = T, T_\parallel, T_\perp;\ p, p_\parallel, p_\perp;\ s, s_\parallel, s_\perp;\ j, j_\parallel, j_\perp)$$

Here, the partial coefficients of heat conductivity of the electrons, ions and neutral particles have the following form:

1) Coefficients of heat conductivity of electrons

$$\begin{gathered} \lambda_e^T = \lambda_e / [1 + (\omega_e \tau_e^*)^2], \quad \lambda_e^{T\parallel} = (\omega_e \tau_e^*)^2 \lambda_e^T, \quad \lambda_e^{T\perp} = - \omega_e \tau_e^* \lambda_e^T \\ \lambda_e^p = - b_e \lambda_e^T, \quad \lambda_e^{p\parallel} = - b_e \lambda_e^{T\parallel}, \quad \lambda_e^{p\perp} = - b_e \lambda_e^{T\perp}, \quad \lambda_e^s = - c_e \lambda_e^T \\ \lambda_e^{s\parallel} = - c_e \lambda_e^{T\parallel}, \quad \lambda_e^{s\perp} = - c_e \lambda_e^{T\perp}, \quad \lambda_e^j = - [d_e - (\omega_e \tau_e^*)^2 d_e'] \lambda_e^T \\ \lambda_e^{j\parallel} = - (d_e + d_e') \lambda_e^{T\parallel}, \qquad \lambda_e^{j\perp} = - (d_e + d_e') \lambda_e^{T\perp} \end{gathered} \tag{2.5}$$

where

$$b_e = \frac{m_i \zeta_{ea} (1-s)}{k(1+Zs)\rho} \varepsilon, \qquad c_e = \frac{\zeta_{ea} (Z+1) T}{Z(1+Zs)s} \varepsilon$$
$$d_e = \frac{m_e m_i}{Z\rho sek} \left(\frac{\zeta_{ei}}{\tau_{ei}} + \frac{\zeta_{ea}}{\tau_{ea}} \right), \qquad d_e' = \frac{m_e m_i}{Z\rho sek} \frac{1-s}{\tau_e^*} \zeta_{ea} \varepsilon \tag{2.6}$$

2) Coefficients of heat conductivity of ions

$$\lambda_i^T = \frac{a_i\vartheta\lambda_i}{1+(\omega_i\tau_i^*\vartheta)^2}, \qquad \lambda_i^{T\parallel} = (\omega_i\tau_i^*\vartheta)^2\,\lambda_i^T, \qquad \lambda_i^{T\perp} = \omega_i\tau_i^*\vartheta\lambda_i^T$$

$$\lambda_i^p = \frac{b_i}{a_i}\lambda_i^T, \quad \lambda_i^{p\parallel} = \frac{b_i}{a_i}\lambda_i^{T\parallel}, \quad \lambda_i^{p\perp} = \frac{b_i}{a_i}\lambda_i^{T\perp}, \quad \lambda_i^s = \frac{c_i}{a_i}\lambda_i^T \tag{2.7}$$

$$\lambda_i^{s\parallel} = \frac{c_i}{a_i}\lambda_i^{T\parallel}, \quad \lambda_i^{s\perp} = \frac{c_i}{a_i}\lambda_i^{T\perp}, \quad \lambda_i^j = -\left[d_i - (\omega_i\tau_i^*\vartheta)^2 d_i'\right]\frac{\lambda_i^T}{a_i}$$

$$\lambda_i^{j\parallel} = -(d_i + d_i')\frac{\lambda_i^{T\parallel}}{a_i}, \qquad \lambda_i^{j\perp} = -(d_i + d_i')\frac{\lambda_i^{T\perp}}{a_i}$$

where

$$a_i = 1 + \frac{\beta\tau_a^*}{\tau_{ia}}, \qquad b_i = \left(a_i - \frac{1}{s}\right)\frac{m_i\zeta_{ia}Z\,s}{2k(1+Zs)\rho}$$

$$c_i = \left(a_i - \frac{1}{s}\right)\frac{\zeta_{ia}(Z+1)\,T}{2(1-s)(1+Zs)}, \qquad d_i = \left(a_i - \frac{1}{s}\right)\frac{m_e m_i\zeta_{ia}}{2ek(1-s)\rho\tau_{ea}} \tag{2.8}$$

$$d_i' = \left(a_i - \frac{1}{s}\right)\frac{m_i^2\zeta_{ia}}{2Zek\rho\tau_i^*\vartheta}, \qquad \vartheta^{-1} = 1 - \frac{\beta^2\tau_i^*\tau_a^*}{\tau_{ai}\tau_{ia}}$$

3) Coefficients of heat conductivity of neutral particles

$$\lambda_a^T = \frac{a_a\vartheta\lambda_a}{1+(\omega_i\tau_i^*\vartheta)^2} + \lambda_a, \quad \lambda_a^{T\parallel} = (\omega_i\tau_i^*\vartheta)^2(\lambda_a^T - \lambda_a)$$

$$\lambda_a^{T\perp} = \omega_i\tau_i^*\vartheta(\lambda_a^T - \lambda_a)$$

$$\lambda_a^p = \frac{b_a(\lambda_a^T - \lambda_a)}{a_a} + \frac{b_a\lambda_a}{a_a - s^{-1}\beta\tau_i^*\tau_{ai}^{-1}}, \qquad \lambda_a^{p\parallel} = \frac{b_a}{a_a}\lambda_a^{T\parallel} \tag{2.9}$$

$$\lambda_a^{p\perp} = \frac{b_a}{a_a}\lambda_a^{T\perp}, \quad \lambda_a^s = \frac{c_a(\lambda_a^T - \lambda_a)}{a_a} + \frac{c_a\lambda_a}{a_a - s^{-1}\beta\tau_i^*\tau_{ai}^{-1}}, \quad \lambda_a^{s\parallel} = \frac{c_a}{a_a}\lambda_a^{T\parallel}$$

$$\lambda_a^{s\perp} = \frac{c_a}{a_a}\lambda_a^{T\perp}, \quad \lambda_a^j = -\left[d_a - (\omega_i\tau_i^*\vartheta)^2 d_a'\right]\frac{\lambda_a^T - \lambda_a}{a_a} - \frac{d_a\lambda_a}{a_a - s^{-1}\beta\tau_i^*\tau_{ai}^{-1}}$$

$$\lambda_a^{j\parallel} = -(d_a + d_a')\frac{\lambda_a^{T\parallel}}{a_a}, \quad \lambda_a^{j\perp} = -(d_a + d_a')\frac{\lambda_a^{T\perp}}{a_a} - \frac{\omega_i\tau_i^*\vartheta\lambda_a d_a'}{a_a - s^{-1}\beta\tau_i^*\tau_{ai}^{-1}}$$

Here

$$a_a = \frac{\vartheta - 1}{\vartheta} + \frac{\beta\tau_i^*}{\tau_{ai}}, \qquad b_a = \left(a_a - \frac{1}{s}\frac{\beta\tau_i^*}{\tau_{ai}}\right)\frac{m_i\zeta_{ia}Zs}{2k(1+Zs)\rho}$$

$$c_a = \left(a_a - \frac{\beta\tau_i^*}{s\tau_{ai}}\right)\frac{\zeta_{ia}(Z+1)\,T}{2(1-s)(1+Zs)} \tag{2.10}$$

$$d_a = \left(a_a - \frac{\beta\tau_i^*}{s\tau_{ai}}\right)\frac{m_e m_i\zeta_{ia}}{2ek(1-s)\rho\tau_{ea}}, \quad d_a' = \left(a_a - \frac{\beta\tau_i^*}{s\tau_{ai}}\right)\frac{m_i^2\zeta_{ia}}{2Z\rho ek\tau_i^*\vartheta}$$

The remaining notation in (2.5), (2.7) and (2.9) is the same as in [10]; in the expressions

$$\lambda_\alpha = {}^5/_2\, km_\alpha^{-1} n_\alpha kT\tau_\alpha^* \; u \; \tau_\alpha^* \qquad (\alpha = e, i, a)$$

n_α should be replaced by ρ, using Equation (1.10).

The coefficients of heat conductivity λ_g^k $(k = p, s, j, j_\perp)$, which characterize the difference $\mathbf{q} - \mathbf{h} = {}^5/_2 \sum n_\alpha kT\mathbf{w}_\alpha$. have the form

$$\lambda_g^p = b_g\lambda_g, \qquad \lambda_g^s = c_g\lambda_g, \qquad \lambda_g^j = d_g\lambda_g, \qquad \lambda_g^{j\perp} = -\,\omega_i d_g'\lambda_g \tag{2.11}$$

where

$$\lambda_g = \frac{5k^2}{2m_i^2}\tau_{ai}\,(1-s)\,T\rho, \qquad b_g = \frac{2m_i Z^2 s^2}{k\,(1+Zs)\,\rho}$$

$$c_g = 2Z\,(Z+1)\,\frac{sT}{(1-s)\,(1+Zs)}, \quad d_g = \frac{m_i^2\tau_{ai}^{-1}}{ek\,(1-s)\,\rho}, \qquad d_g' = \frac{2m_i^2 s}{ek\rho} \tag{2.12}$$

Putting $B = 0$ in (2.5), we obtain, for a partially ionized gas,

$$\mathbf{q} = -\,\lambda^T\,\nabla T - (\lambda^p + \lambda_g^p)\,\nabla p - (\lambda^s + \lambda_g^s)\,\nabla s - (\lambda^j + \lambda_g^j)\,\mathbf{j} \tag{2.13}$$

where, in addition to the ordinary heat conduction in the direction of ∇T (nonionized gas), there appear diffusive heat flows in the directions ∇p, ∇s and $\mathbf{j}$; the term $(\lambda^j + \lambda_g^j)\,\mathbf{j}$ corresponds to the Thomson effect. Comparing (2.3) and (2.13), we see that application of a magnetic field to a partially ionized gas leads to the appearance of additional heat fluxes in directions parallel and transverse to the magnetic field, with the terms $\lambda^T\,(\nabla T \times \mathbf{b})$ and $(\lambda^{j\perp} + \lambda_g^{j\perp})\,(\mathbf{j} \times \mathbf{b})$ corresponding to the Righi-Leduc and Ettinghausen effects. Making use of the formula $\mathbf{k} = \mathbf{b}\,(\mathbf{k}\cdot\mathbf{b}) + \mathbf{b}\times(\mathbf{k}\times\mathbf{b})$, where $\mathbf{k} = \nabla T$, ∇p, ∇s, $\mathbf{j}$, we distinguish three characteristic, mutually perpendicular directions of heat transfer, in which the coefficients of heat conductivity are different. Since the magnetic field does not affect the mean free path of particles along the field, then the coefficient of heat conductivity in that direction is the same as without field

$$\lambda^k + \lambda^{k\parallel} = \lambda^k|_{B=0} \qquad (k = T,\, p,\, s,\, j)$$

On the other hand, the distances traversed without collision by charged particles in a plasma in the transverse direction of the magnetic field, are decreased, and therefore so are the coefficients λ^k and $\lambda^{k\perp}$, characterizing these directions. It is easy to verify that in the isotropic case, when $\omega_e\tau_e^* \ll 1$ and $\omega_i\tau_i^*\vartheta \ll 1$ (i.e. mean free path is equal in all directions), the heat flux vector reduces to the form (2.13).

It should be noted that in the expression for the coefficients of the relative heat flux $\mathbf{h}$, which are connected with the diffusive thermoeffect, the factors ζ_{ea}, ζ_{ia}, ζ_{ei} appear. In [10] it is shown that ζ_{ea} and ζ_{ia} are $\leqslant 0.2$ for real potentials of interaction between molecules and vanish for "Maxwellian" molecules. Therefore, the contribution from the corresponding terms to the heat flux is not large. A somewhat larger contribution is made by the electrons, since the factor $\zeta_{ei} = -0.6$ occurs in λ_e^j, $\lambda_e^{j\parallel}$ and $\lambda_e^{j\perp}$.

We will now evaluate the role of the electrons and ions in the heat flux. Forming the ratio $\lambda_i^k / \lambda_e^k$ ($k = T, T_\parallel, T_\perp; p, p_\parallel, p_\perp; s, s_\parallel, s_\perp; j, j_\parallel; j_\perp$) and neglecting quantities of order $\sqrt{m_e / m_i}$, we can show that, unlike the viscous stress tensor, the heat flux vector in the general case is determined by both the ions and electrons. However, their relative importance is determined by the strength of the magnetic field and the direction of heat flux. Really:

1) if $\omega_e\tau_e^* \lesssim O(1)$

$$\frac{\lambda_i^k}{\lambda_e^k} \sim \begin{cases} \ll 1 & \text{for } k = T, T_\parallel, T_\perp; p_\parallel, p_\perp; s_\parallel, s_\perp; j, j_\parallel, j_\perp \\ O(1) & \text{for } k = p, s \end{cases} \tag{2.14}$$

2) if $\omega_e\tau_e^* = O((m_i / m_e)^{1/4})$

$$\frac{\lambda_i^k}{\lambda_e^k} \sim \begin{cases} \ll 1 & \text{for } k = T_\parallel, T_\perp; p_\parallel; s_\parallel; j_\parallel \\ O(1) & \text{for } k = T; p_\perp; s_\perp; j, j_\perp \\ \gg 1 & \text{for } k = p, s \end{cases} \tag{2.15}$$

3) if $\omega_e\tau_e^* \gg 1$ and $\omega_i\tau_i^*\vartheta \geqslant O(1)$

$$\frac{\lambda_i^k}{\lambda_e^k} \sim \begin{cases} \ll 1 & \text{for } k = T_\parallel \\ O(1) & \text{for } k = T_\perp, p_\parallel, s_\parallel, j_\parallel \\ \gg 1 & \text{for } k = T, p, p_\perp, s, s_\perp, j, j_\perp \end{cases} \tag{2.16}$$

Finally, let us consider the heat flux vector for the particular cases of weakly ionized and fully ionized gases. Putting $s \to 0$ in (2.5), (2.7), (2.9) and (2.11), after some simple calculations we obtain

$$\lambda^k = \begin{cases} [\lambda_a]_{s=0} + O(s) & \text{for } k = T \\ O(s) \quad \text{for } k = T_\parallel, T_\perp, p, p_\parallel, p_\perp & \left([\lambda_a]_{s=0} = \dfrac{75k^2T}{32m_a\Omega_{aa}^2(2)} = \dfrac{5}{2}\,\dfrac{3k}{2m_a}\eta_a \right) \\ \sum\limits_{\alpha=e,i,a,g} [\lambda_\alpha^k]_{s=0} + O(s) & \text{for } k = s, s_\parallel, s_\perp, j, j_\parallel, j_\perp \end{cases} \tag{2.17}$$

This coincides with the coefficient of heat conductivity of a simple gas according to Chapman-Cowling [12].

Putting $s = 0$ in (2.5), (2.7) and (2.11) and writing (2.3) in the special coordinate system with z-axis parallel to the vector $\mathbf{b}$, we find for the heat flux vector of a fully ionized gas an expression which is the same (up to numerical factors) as that given in [7 and 8].

3. System of equations of anisotropic magnetogasdynamics. The equations of conservation of mass, momentum and energy in the one-fluid approximation, within the restrictions set by our assumptions, have the form

$$\frac{\partial\rho}{\partial t} + \operatorname{div} \rho\mathbf{u} = 0 \tag{3.1}$$

$$\rho\frac{d\mathbf{u}}{dt} = -\nabla p - \operatorname{div}\pi + \mathbf{j}\times\mathbf{B} \qquad \left(\frac{d}{dt} = \frac{\partial}{\partial t} + (\mathbf{u}\nabla)\right) \tag{3.2}$$

$$\frac{3}{2}\frac{dp}{dt} + \frac{5}{2}p\operatorname{div}\mathbf{u} = -\operatorname{div}\mathbf{q} - \pi^{rm}\frac{\partial u^r}{\partial x_m} + \mathbf{j}(\mathbf{E} + \mathbf{u}\times\mathbf{B}) \tag{3.3}$$

Here E is the electric field intensity vector; in place of π^{rs} and $\mathbf{q}$ the expressions (1.4) and (2.3) should be substituted. The equation of state is given by kinetic theory in the form $p = nkT$. Taking into account the relation (1.10) between the number density n and mass density of gas, we have

$$p = \frac{1+Zs}{\mu_a}\rho RT = (1+Zs)\rho R' T \qquad \left(R' = \frac{R}{\mu_a}\right) \tag{3.4}$$

Here $\mu_a \approx \mu_i$ is the "molecular weight" of the neutral (or ion) gas, R is the gas constant per gram-molecule. Using (3.4), (3.1), and the specific heat at constant volume of the α-gas, $c_{v_\alpha} = {}^3/_2\,(k/m_\alpha)$, as determined in [12], the energy equation (3.3) may be put in the form

$$(1+Zs)\,c_{v_a}\rho\frac{dT}{dt} + Zc_{v_a}\rho T\frac{ds}{dt} =$$
$$= -p\operatorname{div}\mathbf{u} - \operatorname{div}\mathbf{q} - \pi^{rm}\frac{\partial u^r}{\partial x_m} + \mathbf{j}(\mathbf{E}+\mathbf{u}\times\mathbf{B}) \tag{3.5}$$

Introducing the heat energy per unit mass of the α-component of the mixture, $\varepsilon_\alpha = c_{v_\alpha}T$, we obtain

$$(1+Zs)\rho\frac{d\varepsilon_a}{dt} + Z\rho\varepsilon_a\frac{ds}{dt} = -p\operatorname{div}\mathbf{u} - \operatorname{div}\mathbf{q} - \pi^{rm}\frac{\partial u^r}{\partial x_m} + \mathbf{j}(\mathbf{E}+\mathbf{u}\times\mathbf{B}) \tag{3.6}$$

The local degree of ionization $s(x, y, z, t)$ appearing in (1.4), (2.3), (3.3) and (3.4) may be found from the continuity equation for the α-component of the partially ionized gas

$$\frac{\partial n_\alpha}{\partial t} + \operatorname{div}(n_\alpha\mathbf{w}_\alpha + n_\alpha\mathbf{u}) = 0 \tag{3.7}$$

Taking into account (1.10), (2.1) and (3.1), we obtain

$$\rho\frac{ds}{dt} - \frac{m_e}{e}\mathbf{j}\nabla s + \operatorname{div}\rho s(1-s)\mathbf{V}_i = 0 \tag{3.8}$$

Here, the ion "slip" velocity $\mathbf{V}_i$ is given by Expression (2.2).

To the system of equations written above, it is necessary to add the equations of electrodynamics,

$$\operatorname{rot}\frac{\mathbf{B}}{\mu_0} = \mathbf{j} + \frac{\partial\varepsilon_0\mathbf{E}}{\partial t}, \quad \operatorname{rot}\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}; \quad \operatorname{div}\mathbf{B} = 0, \quad \operatorname{div}\varepsilon_0\mathbf{E} = \rho_e \tag{3.9}$$

as well as the generalized Ohm's law, which, with an accuracy up to quantities of order $\sqrt{m_e/m_i}$ is given by Equation (4.10) in [10]. It is also necessary to take into account the influence of viscous momentum transfer on the diffusion of charged particles.

Using one-fluid parameters, we have

$$\sigma^j \mathbf{j} + \sigma^{j\,\|}\, \mathbf{b}\,(\mathbf{j}\cdot\mathbf{b}) + \sigma^{j\perp}\,(\dot{\mathbf{j}}\times\mathbf{b}) =$$
$$= \sigma_0\,[\mathbf{E} + \mathbf{u}\times\mathbf{B} + \sigma^T\nabla T + \sigma^{T\,\|}\,\mathbf{b}\,(\nabla T\cdot\mathbf{b}) + \sigma^{T\perp}(\nabla T\times\mathbf{b}) + \sigma^p\nabla p +$$
$$+ \sigma^{p\perp}(\nabla p\times\mathbf{b}) + \sigma^s\nabla s + \sigma^{s\perp}(\nabla s\times\mathbf{b}) + \sigma^{\pi\perp}(s\,\mathrm{div}\,\pi - \mathrm{div}\,\pi_i)\times\mathbf{b}] \quad (3.10)$$

where

$$\sigma_0 = \frac{n_e e^2\tau_0}{m_e} = \frac{Ze^2}{m_e m_i} s\rho\tau_0, \qquad \sigma^j = 1 - \frac{\Delta_0}{1+(\gamma\omega_e\tau_0)^2} + \delta_0\,(\omega_e\tau_0)^2$$

$$\sigma^{j\,\|} = -\,\delta_0\,(\omega_e\tau_0)^2 - \frac{\Delta_0\,(\gamma\omega_e\tau_0)^2}{1+(\gamma\omega_e\tau_0)^2}, \qquad \sigma^{j\perp} = \omega_e\tau_0 + \frac{\Delta_0\gamma\omega_e\tau_0}{1+(\gamma\omega_e\tau_0)^2} \quad (3.11)$$

$$\sigma^T = -\frac{\alpha_T k/e}{1+(\gamma\omega_e\tau_0)^2}, \qquad \sigma^{T\,\|} = (\gamma\omega_e\tau_0)^2\,\sigma^T, \qquad \sigma^{T\perp} = -\,\gamma\omega_e\tau_0\sigma^T$$

$$\sigma^p = \frac{m_i}{e\,(1+Zs)\,\rho}, \qquad \sigma^{p\perp} = -\,\delta_0\omega_e\tau_0\sigma^p, \qquad \sigma^s = \frac{m_i p}{e\,(1+Zs)^2 s\rho}$$

$$\sigma^{s\perp} = -\,\frac{Z+1}{Z}\,\frac{\delta_0\omega_e\tau_0}{1-s}\,\sigma^s, \qquad \sigma^{\pi\perp} = \frac{m_i\delta_0\omega_e\tau_0}{Ze\,(1-s)\,s\rho}$$

The other notation is the same as in [10].

Thus, we have obtained a closed system of 17 equations (3.1) to (3.4), and (3.8) to (3.10), with 17 unknowns $\mathbf{u}$, $\mathbf{B}$, $\mathbf{E}$, $\mathbf{j}$, p, ρ, s, T and ρ_e, to describe the dynamics of a partially ionized gas in a magnetic field.

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